CALCULUS AB

SECTION II

Time — 1 hour and 30 minutes

Number of problems — 6

Percent of total grade — 50

SHOW ALL YOUR WORK. Indicate clearly the methods you use because you will be graded on the correctness of your methods as well as on the accuracy of your final answers. If you choose to use decimal approximations, your answer should be correct to three places after the decimal point.

Note: Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. Let f be the function given by $f(x) = 3x^4 + x^3 - 21x^2$.

   (a) Write an equation of the line tangent to the graph of f at the point $(2, -28)$.

   (b) Find the absolute minimum value of f. Show the analysis that leads to your conclusion.

   (c) Find the x-coordinate of each point of inflection on the graph of f. Show the analysis that leads to your conclusion.

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2. Let R be the region enclosed by the graphs of $y = e^x$, $y = x$, and the lines $x = 0$ and $x = 4$.

   (a) Find the area of R.

   (b) Find the volume of the solid generated when R is revolved about the x-axis.

   (c) Set up, but do not integrate, an integral expression in terms of a single variable for the volume of the solid generated when R is revolved about the y-axis.

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3. Consider the curve defined by $x^2 + xy + y^2 = 27$.

   (a) Write an expression for the slope of the curve at any point (x, y).

   (b) Determine whether the lines tangent to the curve at the x-intercepts of the curve are parallel. Show the analysis that leads to your conclusion.

   (c) Find the points on the curve where the lines tangent to the curve are vertical.

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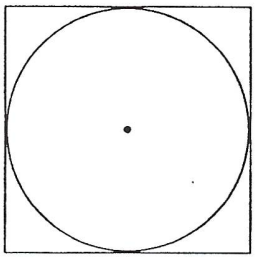
4. A particle moves along the x-axis so that at any time $t > 0$ its velocity is given by $v(t) = t \ln t$. At time $t = 1$, the position of the particle is $x(1) = 6$.

   (a) Write an expression for the acceleration of the particle.

   (b) For what values of t is the particle moving to the right?

   (c) What is the minimum velocity of the particle? Show the analysis that leads to your conclusion.

   (d) Write an expression for the position $x(t)$ of the particle.



5. A circle is inscribed in a square as shown in the figure above. The circumference of the circle is increasing at a constant rate of 6 inches per second. As the circle expands, the square expands to maintain the condition of tangency. (Note: A circle with radius r has circumference $C = 2\pi r$ and area $A = \pi r^2$.)

   (a) Find the rate at which the perimeter of the square is increasing. Indicate units of measure.

   (b) At the instant when the area of the circle is 25π square inches, find the rate of increase in the area enclosed between the circle and the square. Indicate units of measure.

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6. Let $F(x) = \int_0^x \sin(t^2)dt$ for $0 \le x \le 3$.

   (a) Use the trapezoidal rule with four equal subdivisions of the closed interval [0, 1] to approximate $F(1)$.

   (b) On what intervals is F increasing?

   (c) If the average rate of change of F on the closed interval [1, 3] is k, find $\int_1^3 \sin(t^2)dt$ in terms of k.

END OF EXAMINATION